MATHEMATICS, Undergraduate Study Programme, first Bologna cycle
COURSE DESCRIPTIONS (MA-17)

TABLE OF CONTENTS

1 COMPULSORY COURSES

1.1 Course name: ALGEBRA I – MATRIX CALCULUS

Number of ECTS credits: **6**

Content:
- Vectors, analytic geometry in space.
- Matrices. Types of matrices and basic operations with matrices. Rank of a matrix. Inverse.
- Systems of linear equations. Matrix interpretation and theorem of solvability. Elementary matrices, Gauss method. Determinants. Cramer's rule.

1.2 Course name: ALGEBRA II – LINEAR ALGEBRA

Number of ECTS credits: **6**

Content:
- Groups, rings,domains. Polynomial rings.
- Vector space. Subspaces, linear operators. Linear independence. Basis and the dimension vector space.
- Eigenvalues. The characteristic and minimal polynomial.
- Inner product. Orthogonal systems. Gramm-Schmidt orthogonalization process. Norm. Norm of the matrix and the operator. Normal and related operators.
- Convexity in the vector space.
- Normed vector spaces as metric spaces. Isometries of R2 and R3.

1.3 Course name: ANALYSIS I – THE FOUNDATIONS OF ANALYSIS

Number of ECTS credits: **6**

Content:
- Natural numbers. Rational numbers. Real numbers. Complex numbers.
- Sequence of real numbers. Limits and accumulation points. Cauchy condition. Upper and lower limit. Monotone sequences. Bolzano-Weierstrass theorem.
- Series. The convergence criteria. Absolutely and conditionally convergent series.
- Functions of real variables, even and odd functions, periodicity. Limits of functions, left and right limits. Continuity. Continuous functions on closed intervals limited. Bisection method for finding zeros.
- Elementary functions. Cyclometric functions.

1.4 Course name: ANALYSIS II – INFINITESIMAL CALCULUS

Number of ECTS credits: **6**

Content:
- Derivative. Mean value theorems. Differentiation of monotone functions. L'Hopital's rule. Higher derivatives. Taylor's formula. Local extrema. Convex and concave functions. Inflection points. Tangent method of finding the zeros.
- The indefinite integral. Definite Integrals. Darboux and Riemann sums. Leibniz-Newton formula. Mean value theorems. Integration methods. Applications of the definite integral in geometry. Improper integral. Numerical integration.
- The logarithm, the number e, and the definition of exponentiation with the real exponent.
- Drawing planar curves.
- Sequences and function series. Power series. Taylor series. Elementary
- complex functions.

1.5 Course name: DISCRETE MATHEMATICS II – COMBINATORICS

Number of ECTS credits: **6**

Content:

1) Mandatory topics (approximately 70%-75% of contact hours):

- Introduction. What is combinatorics? Sample problems (such as: derangements, Euler's officers, Kirkman's schoolgirls, a Ramsey game etc.).

- Basic Combinatorial Principles. The addition principle. The double counting principle. The multiplication principle. The equality principle. The pigeonhole principle (Dirichlet's principle). Generalized Pigeonhole Principle.

- Elementary Combinatorics. Ordered selections with repetition. Ordered selections without repetition (special case: permutations). Unordered selections without repetition, formula for binomial coefficients. Quotient representation of binomial coefficients. Unordered selections with repetition. Permutations with repetition. Pascal's identity and Pascal's triangle. Binomial Theorem and consequences. Principle of Inclusion-Exclusion. Counting derangements. Counting surjective functions.

- Recurrence Relations. Examples of recurrence relations. Fibonacci numbers. Linear recurrence relations with constant coefficients. Derangements, revisited.

- Distributions. Distributing labeled elements into unlabeled cells. Stirling numbers of the second kind, recurrence relation and relation to Bell numbers. Distributing labeled elements into labeled cells. Distributing unlabeled elements into unlabeled cells. Partitions of a positive integer n of order k. Distributing unlabeled elements into labeled cells.

- Graphs. Basic definitions. Graph isomorphism. Walks, trails, paths, connected components. Trees and forests: definitions and basic properties. Spanning trees. Minimum Spanning Trees: Kruskal's Algorithm. Eulerian Graphs: Euler's Theorem. Hamiltonian graphs. Hall's Theorem, systems of distinct representatives. Application: Latin squares. Graph vertex- and edge colorings. Planar graphs.

2) For the remaining 25%-30% of contact hours, the instructor shall, at his / her discretion, select topics from the following list:

2.1) Additional topics from above chapters:

- Recurrence Relations. Formal power series. Generalized Binomial Theorem. Catalan and Bell numbers.

- Graphs. Cayley's Theorem. The Traveling Salesman Problem. Digraphs. Shortest paths, Dijkstra's algorithm. Tournaments. Kőnig-Egérvary's theorem. Menger's theorem. Chromatic polynomial of a graph. Graph homomorphisms. Ramsey's theorem.

2.2) Topics from additional chapter:

- Introduction to Designs. Balanced Incomplete Block Designs - definition and examples. Steiner triple systems. Incidence matrix of a design. Fisher's inequality. Symmetric designs. Resolvable Designs. Solution to Kirkman's schoolgirl problem.
Finite projective planes, affine planes. Difference methods. Hadamard designs and Hadamard matrices.

1.6 Course name: MATHEMATICAL PRACTICUM I

Number of ECTS credits: **6**

Content:

- Presentation software (eg PowerPoint), spreadsheets (eg Excel)
- Text editors (eg WinEdt, TextPad, Emacs, Auctech, Open Office ...)
- Basics of TeX and LaTeX (MikTeX, TeTex, GSview, Acrobat Reader ...)
- The basic image tools (pdf, eps), working with image formats, including images in LaTeX, scanning and the use of digital cameras

1.7 Course name: COMPUTER PRACTICUM

Number of ECTS credits: **6**

Content:
The course will cover topics that might include (but are not restricted to) any of the following according to the needs and development of the subjects covered:

- Hardware.
- The structure of a computer system, the memory hierarchy, devices, bus, I/O devices, CPU. Operating system. Processes, synchronisation, devices, process scheduler, device managers.
- OS Linux basics.
- The Linux OS and its Slovenian flavour – Pingo Linux. Usage of the BASH shell.
- Programming languages.
- Programming language types. Imperative, object-oriented, logical, declarative programming languages. Concepts of programming languages. Iteration, data structures, control structures, functions, subroutines. Basic data structures: fields, arrays and lists. The C programming language.
- The object oriented model.
- The concepts of the object oriented model: objects, classes, fields, methods, inheritance, polymorphism, multiple inheritance, interfaces and abstract classes. Abstract data types. Examples of abstract data types. The Java programming language.

1.8 Course name: COMPUTER SCIENCE I

Number of ECTS credits: **6**

Content:
The course will cover topics that might include (but are not restricted to) any of the following according to the needs and development of the subjects covered:

- Basic building blocks of a computer program (using the syntax of the programming language Java): Variables, types and expressions. Basic I/O operations. Decision statements. Control structures. Functions and parameters. Programs. Structural decomposition.

- Basic data structures:
Simple types. Arrays. Records. Characters and strings. Data representation in computer memory. Memory allocation. Linked structures. Stack. Queue. List. Tree.

- Algorithms and problem solving:
What is an algorithm? Problem solving strategies. The role of algoirithms in problem solving. Algorithm implementation strategies. Debugging. Recursion – recursive functions, divide-and-conquer principle, backtracking, implementation of recursion.

- Programming languages overview:
Types of programming languages. Flow control. Functions. Subprograms. Namespaces.

- Declarations and types:
Types. Declarations of types. Safe typing. Type checking. Subtypes. Classes. Polymorphism.

- Abstraction mechanisms:
Data abstractions. Simple types. Composite types. Flow abstractions. Subprograms and functions. Abstract data types. Objects and classes. Patterns. Modules.

1.9 Course name: DISCRETE MATHEMATICS I – SET THEORY

Number of ECTS credits: **6**

Content:
- Fundamentals of mathematical theory, propositional calculus, truth tables, predicate calculus.
- Formal languages.
- Basic concepts of mathematical logic.
- Ways of describing sets. Basic relations between sets, basic operations on sets or families of sets. Power set. Relations. Graphs. Equivalence relations. Partial and linear ordering. Lattices and Boolean algebra. Well-ordered sets. Functions. Special types of functions. Categories.
- Finite and infinite, countable and uncountable sets.
- Cardinal and ordinal numbers. Peano arithmetics, mathematical induction.
- NBG and ZFC systems of axioms of set theory. Axiom of choice. Zorn's lemma.
- Fundamentals of symbolic computation (Mathematica).

1.10 Course name: MATHEMATICAL TOPICS IN ENGLISH I

Number of ECTS credits: **6**

Content:
The classical as well as most current research topics in the field of mathematics are taught, which, among others, may include the following topics:
- Algebra,
- Analysis,
- Discrete mathematics,
- Financial mathematics,
- Cryptography,
- Computer intensive methods and applications,
- Linear programming,
- Statistics.

1.11 Course name: ALGEBRA III – ABSTRACT ALGEBRA

Number of ECTS credits: **6**

Content:
- Introduction to number theory, Euclidean algorithm, congruences.
- Polynomials o in single variable. Euclidean algorithm. Zeros of polynomials. Solving algebraic equations. Polynomials in several variables. Symmetric polynomials. Fundamental theorem of algebra.
- Grupoids, semigroups and groups. Homomorphisms of groups. Normal subgroups and factor groups. Families of groups. Groups given by generators and relations. Sylow theorems.

1.12 Course name: ANALYSIS III – FUNCTIONS OF MANY VARIABLES

Number of ECTS credits: **6**

Content:
- Metric spaces. Cauchy-Schwarz inequality. Open and closed sets. Compactness and connectedness. Sequences in metric spaces. Cauchy sequences and complete spaces. Continuity and uniform continuity. Properties of continuous mappings.
- Functions of several variables. Continuity, partial differentiability. Differential. Mappings from R^n to R^m. Jacobian matrix. Chain rule.
- Higher order partial derivatives. Taylor's formula. Theorem on the inverse and implicit functions. Local extrema and constrained local extrema
- Double and Multiple integrals. Properties. Theorems on existence. The substitution of new variables. Calculation and application.
- Proper and improper integrals with the parameter.

1.13 Course name: PHYSICS

Number of ECTS credits: **6**

Content:
- Physical measurments
- Linear motion
- Movement in three dimensions
- Forces and motion (Newton's laws, friction)
- Kinetic energy and work
- Potential energy, conservation of energy
- Systems of particles (centre of mass, momentum)
- Rotation
- Rotational quantity
- Equilibria and elastic properties
- Gravitation
- Fluid mechanics
- Oscillation
- Waves (general properties, types of waves, sound)
- Heat (temperature, laws of thermodynamics, heat conductivity)
- Kinetic theory of gasses
- Entropy
- Electric charge
- Electric field
- Electric potential
- Capacity
- Electric current resistance
- Magnetic field
- Induction
- Alternating current and electromagnetic oscillations
- Electromagnetic waves
- Geometrical optics
- Interference and diffraction
- Basics of modern physics (Photons and material waves, material waves, atom physics, atom core, special relativity theory)

1.14 Course name: INTRODUCTION TO NUMERICAL CALCULATIONS

Number of ECTS credits: **6**

Content:

- Fundamentals of numerical computa-tion. Floating point computation and numerical error. Stable computational processes and the sensitivity. The total error.
- Nonlinear equations. Bisection. Iteration methods. Tangent method, secant method. Algebraic equations. Systems of nonlinear equations. Newton's method.
- Systems of linear equations. LU and Cholesky decompositions. Gauss elimination. Diagonally dominant and tridiagonal matrices. Condition number.
- Solving linear systems iteratively.
- Least squares problem. Predetermined systems. QR decomposition. SVD decomposition.
- Eigenvalues of matrices. Power method and inverse power method. Gerschgorin's and Schur's theorem.
- Interpolation of functions. Polynomial interpolation. Divided differences. Interpolation with splines.
- Numerical derivation. Numerical integration. Newton-Cotes rules. Composed rules. Gauss quadratures. Euler-Maclaurin formula and Romberg extrapolation. Multivariate integration and cubature rules.
- Bezier curves. De Casteljau algorithm. Bernstein polynomials. Properties of Bezier curves. Subdivision. Degree elevation.

1.15 Course name: COMPUTER SCIENCE II

Number of ECTS credits: **6**

Content:
The course will cover topics that might include (but are not restricted to) any of the following according to the needs and development of the subjects covered:

- Introduction

Introduction to programming languages, concepts of programming languages, Meta-language, Chomski hierarchy, computability, overview of programming language history.

- Lambda calculus

History of λ-calculus, λ-abstraction, definition of λ-calculus, evaluation, substitution, alpha reductions, beta reductions, programming in λ-calculus, Church numbers, recursion, uses of λ-calculus.

- Syntax

Grammars, parsing, parse trees, BNF, grammar definition, operator, priority of operator, asociativity, dangling else, abstract syntax tree, BNF variations.

- Basic structures

Values, basic types, variable declaration, global declaration, local declaration, implementation of variables, symbol tables, name-spaces.

- Functional languages

Mathematical and logic foundations, function expressions, function definition, recursive functions, polymorphism, higher-order functions, examples of functions.

- Imperative languages

Variables, sequential control, structured control, if statement,loops, patterns, function implementation, parameters, activation records, array, functions on arrays.

- Types

Introduction to types, type declaration, products, records, unions, vectors, recursive types, parametrized types, type checking, type inference, examples of use of types.

- Modules

Modules as units of compilation, interface and implementation, separate compilation, language of modules, information hiding, sharing types among modules, functors, examples of module implementations.

- Objects and classes

Introduction to object-oriented languages, object logic, class definition, aggregation, specialization, inheritance, self and super, object initialization, method overloading, dynamic binding, abstract classes, polymorphism, parametrized classes, introspection, exceptions, implementation of classes and objects.

1.16 Course name: PROBABILTY

Number of ECTS credits: **6**

Content:

Basic combinatorics

-Rule of product, rule of sum.
-Variations and variations with repetition.
-Combinations and combinations with repetition.
-Permutations and permutations with repetition.
-Binomial formula with its generalizations.

Outcomes and events

-Sample space, events, probability measure.
-Computations with events.
-Conditional probability and independence.

Random variables

-Random variables and their distributions.
-Basic discrete distributions.
-Continuous random variables.

Multivariate distributions

-Definition of discrete multivariate distributions.
-Discrete multivariate distributions.
-Multivariate continuous distributions.
-Conditional distributions and independent random variables.

Expectation and variance

-Expectation and its properties
-Variance and covariance
- Conditional expectation

Generating functions

-Definition and examples.
-Branching processes.

Approximation of distributions

-Convergence in Distribution.
-Normal approximation of a sum of random variables.
-Poisson's approximation.

1.17 Course name: MATHEMATICAL TOPICS IN ENGLISH II

Number of ECTS credits: **6**

Content:
The most current research topics in the field of mathematics are taught, which, among others, may include the following topics:
- history of the concept of number,
- number theory,
- algebra,
- analysis,
- linear programming,
- famous construction problems,
- overview of the history of computing,
- history of Mathematics in Slovenia,
- historical development of mathematical concepts.

1.18 Course name: ALGEBRA IV – ALGEBRAIC STRUCTURES

Number of ECTS credits: **6**

Content:
- Rings. Ideals. Homomorphism of rings. Factor rings. Integral domain. Euclidean rings. Principal ideal rings. Gaussian coils. Gaussian integers. Chinese remainder theorem.
- Fields. Subfields. Extensions. Final extension.
- The rate of extension. The degree theorem. Simple algebraic extension. Splitting field.
- Constructions with ruler and compass. Squaring the circle. Angle trisection. Doubling Cube. Constructions of regular polygons.

1.19 Course name: ANALYSIS IV – REAL ANALYSIS

Number of ECTS credits: **6**

Content:
- Fourier series. Bessel inequality of vector spaces with inner product. Orthonormal system and orthonorm basis. Fourier integral and Fourier transform.

- Differential geometry of curves in the plane and space. The length of the curve. Natural parameter. Frenet formulas.Surfaces. Curvilinear cordinates.Tangent plane. The first fundamental form. Area of the surface.

- Vector analysis. Scalar and vector fields. Gradient, divergence, curl. Potential and solenoid field. Line integrals and surface integrals of the first and second type. Gauss and Stokes theorem.

1.20 Course name: STATISTICS

Number of ECTS credits: 6

Sampling:
- The concept of random sampling
- Sampling distribution and standard error
- Examples of sampling and their standard errors
- Stratified sampling and examples of allocations

Parameter estimation:
- The concept of a statistical model

- Parameter space, estimators, sampling distribution
- Maximum likelihood method
- Asymptotic properties of the maximum likelihood method
- Rao-Cramér inequality, optimality of estimates, factorization theorem

Hypothesis testing:
- Problem formulation
- Statistical tests, test size, power of tests
- Examples of statistical tests
- Wilks' Theorem
- Neyman-Pearson lemma, theory of optimality

Linear models:
- Assumptions of linear models and examples
- Parameter estimation
- Gauss-Markov theorem
- Generalizations of linear models
- Applications

1.21 Course name: MATHEMATICAL MODELLING

Number of ECTS credits: **6**

Content:
- *Introduction.* What is mathematical modeling? The role of mathematical models in natural sciences and economics. Types of mathematical models.
- *Programming tools*. A short overview of Octave/Scilab.
- *Optimization.* Critical point, minimum, maximum, saddle. Taylor's formula for scalar fields. Local extrema and local extrema under constraints. Newton's method. Applications: discrete catenary, truss stability etc.
- *Calculus of variations.* Standard problem of variation calculus. Isoperimetric problems. Applications: catenary, brachistochrone, truss oscillations, etc.
- *Linear programming*. What is a linear program? Examples of linear programs: optimal diet, flow in a network etc. Forms of linear programs. The fundamental theorem of linear programming. Simplex method. Duality. Integer linear programming and LP relaxation. Applications.
- *Differential equations and systems of differential equations as mathematical models in natural sciences*. Motivational examples. Equilibrium. (Linear) Stability of equilibria. Phase portraits. The basics of Poincare-Bendixon theory. The basics of bifurcation theory. Applications: epidemic models, models of competition, models of symbiosis, predator-prey dynamics, molecular kinetics, basic neurological models, models in economics.

2 ELECTIVE COURSES

2.1 Course name: ALGEBRAIC GRAPH THEORY

Number of ECTS credits: **6**

Content:
- Automorphism group of the graph
- Symmetries of graphs
- Graphs with transitive automorphism group (vertex-transitive graphs, edge-transitive graphs, arc-transitive graphs, distance transitive graphs)
- Strongly regular graphs

2.2 Course name: DIFFERENTIAL EQUATIONS

Number of ECTS credits: **6**

Content:
- Differential Equations. Examples from geometry and physics. Cauchy problem and Euler's method of solution.
- The elementary integration methods for ordinary differential equations. Existence theorems. Differential equations of higher orders. Linear differential equations. Systems of differential equations. Separable variables. Homogeneous right-hand side. Linear equation. Bernoulli and Riccati equatin.
- Calculus of variations. The basic problem of calculus of variations. Euler's equation. Isoperimetric problem.
- Bessel differential equation. Solution with the series. Representation with series and integrals.
- Numerical solutions.
- Laplace transform. Inverse formula, properties. Application.
- Boundary problems for differential equations of second order. Sturm-Liouville operator

2.3 Course name: FUNCTIONAL ANALYSIS

Number of ECTS credits: **6**

Content:
- Linear topological spaces
- Normed and Banach spaces
- Bounded operators
- Hahn-Banach theorem
- Separation theorem of convex
- Baire theorem
- Open mapping theorem
- Uniform boundedness principle
- Closed graph theorem
- Spaces with scalar product and Hilbert spaces
- Riezs representation theorem of a bounded linear functional
- Adjoint of an operator
- Selfadjoint, unitary, and normal operators
- Spectrum of an operator
- Compact operators

2.4 Course name: SELECTED TOPICS IN DISCRETE MATHEMATICS

Number of ECTS credits: **6**

Content:
- Association schemes: definition, basic properties, examples, intersection numbers.
- Bose-Mesner algebra: basis, properties.
- Primitive idempotents: definition, Krein parameters.
- Distance-regular graphs: definition, examples, intersection numbers.
- Some necessary conditions for the existence of distance-regular graphs with prescribed intersection numbers.
- Primitive and imprimitive distance-regular graphs.

2.5 Course name: SELECTED TOPICS IN FROM STATISTICS

Number of ECTS credits: **6**

Content:
Time series analysis:
- Time series
- Stationary processes
- ARMA models
- Parameter estimation
- Model testing
- Time series and forecasting
- ARCH models and its variations.

2.6 Course name: COMBINATORICS

Number of ECTS credits: **6**

Content:
- Basic methods of combinatorics: Classification of discrete problems, basic rules of combinatorics, Selections, Inclusion and exclusion principle, generating functions, rook polynomials
- Combinatorics and recursion: Distributions, Polynomial sequences, Descending powers, Stirling number of first and second kind, Lah numbers and antidifferences, Sums, linear recursion
- Theory of discrete probability, experiment, event, conditional probability, independence, Relay experiments, random variables, Mathematical expectation and variance.

2.7 Course name: COMPLEX ANALYSIS

Number of ECTS credits: **6**

Content:
- A complex plane. The extended plane and stereographic projection. Power series with complex arguments. Exponential function. Logarithmic function and root functions.
- Differentiation of complex functions. Cauchy-Riemann equations. Entire functions.
- Integration of complex functions along the path. Cauchy- theorems. Morera's theorem. Liouville's theorem and the fundamental theorem of algebra. The principle of maximum modulus. Homotopy.
- Isolated singularities. Laurent series. Residues and its applications.
- Harmonic functions. Poisson kernel and Poisson integrals. Solution of Dirichlet problem on the circle. Harnack's theorem. Average value property and harmonic functions. Subharmonic functions.
- Schwarz's Lemma. Principle of maximum modulus. Rado's theorem.
- Approximation of rational functions. Runge's theorem. Conformal mappings. Normal family. Riemann theorem on the conformal equivalence.
- Infinite products. Zeros of holomorphic mappings. Weierstrass factorization theorem. Meromorphic functions and Mittag-Leffler's theorem.
- Jensen's formula. Blaschke products and functions in H ∞.

2.8 Course name: GEOMETRY

Number of ECTS credits: **6**

Content:
- Axiomatics of Euclidean and hyperbolic geometry,
- metric, Pasch, protractor, and absolute geometry,
- Theory of parallelism in absolute geometry (Khayyam-Saccheri's and Lambert 's quadrilateral, Saccheri–Legendre theorem),

- Fundamental properties of hyperbolic plane (critical function, asymptotic parallelism, defect of triangles, isometries, different models),
- fundamentals of convex geometry of Euclidean space of higher dimensions (Charateodory theorem, theorems on separations, convex polytopes, Euler-Poincare formula).

2.9 Course name: CRYPTOGRAPHY AND COMPUTER SAFETY

Number of ECTS credits: **6**

Content:

- Classical ciphers and hoistorical development,
- Fiestel's cipher and AES (Advanced Encryption Standard),
- Finite fields and Extended Euclidean algorithm,
- Public crypto systems, one-way functions and related problems from number theory (testing primality, factorization of integers, discrete logarithm problem),
- Hash functions and message integrity (authentication),
- Key exchange protocols and identification protocols,
- Pseudo random number generator,
- Other protocols (flipping a coin over the telephone, mental poker, secret sharing, verification, codes, vizual cryptography, zero knowledge proofs)
- Public key infrastructure (PKI), certificate authority (CA),
- Broader view on cryptography – security of information and network security.

2.10 Course name: MATHEMATICS: METHODS AND ART

Number of ECTS credits: **6**

Content:

- Generating mathematical truths.
- Mathematics: method and art. Numbers 1, 2, 3, 5, 7 and basic principles of thinking. Real and virtual. Restriction, extension and symmetry. Mathematization of science.
- Mathematics in natural sciences, social sciences, arts and politics. Concrete examples: Parliamentary elections and geometric configurations; Genome, Chinese I-Ching and the hypercube; symmetries of molecular graphs and fullerenes; Sports tournaments and graph manipulation; Albrecht Durer - Melancholy, truncated cube, and Pappus configuration; Durer and magic squares. Primes, factorization, and secret codes.

2.11 Course name: MOLECULAR MODELING

Number of ECTS credits: **6**

Content:

- The concepts of molecular modelling
- Introduction to Classical and Quantum Mechanics
- Potential field and molecular mechanics
- Computer simulation methods
- Molecular dynamics simulations
- Monte Carlo methods
- Using molecular modeling techniques in chemistry, pharmacy, biophysics, etc..

2.12 Course name: OPTIMIZATION METHODS

Number of ECTS credits: **6**

Content:

1) Linear programming.
- Brief introduction to convexity and polyhedra.
- Examples for LP problems, model setup, geometric interpretation.
- Simplex method, revised simplex method, two-phase simplex method.
- Duality (Farkas' lemma, hyperplane separation).
- Algorithmic issues: finiteness, cycling, optimality, LU-factorization.
- Matrix games*.
- Continuous knapsack*, Dantzig's solution*.
- Integer knapsack, dynamic programming solution.
- Cutting stock problems, column/row generation.
- Problem size, complexity of algorithms, polynomial-time algorithms.
- Ellipsoid method and the separation routine, the Grőtschel-Lovász-Schrijver theorem (with no proof).
- Network simplex*, transshipment* and transportation problems.*

2) Discrete optimization and complexity.
- Shortest paths (Formulation as LP and as min-cost flow problem, Dijkstra's algorithm, Bellman-Ford equations, connection with dynamic programming).
- Maximum flow algorithms (path and arc-flow formulations, augmenting paths, Ford-Fulkerson algorithm, max-flow min-cut theorem; link with LP duality; Dinitz, Edmonds-Karp improvements).
- Bipartite matching (unweighted case, integer linear programming formulation, max flow formulation, alternating path algorithm; weighted case, assignment problem, Hungarian algorithm).
- Spanning trees (greedy algorithm).
- Complexity (introduction to NP: decision problems, polynomial transformations, NP-completeness; satisfiability problem and Cook's theorem; some examples of polynomial transformations and NP-completeness proofs; “equivalence" of optimization and decision versions, solution of optimization problems by binary search).
- Approximation algorithms (concepts, vertex cover, spanning tree algorithm for the metric TSP (possibly including a discussion of Eulerian tours), proof that TSP without triangle inequalities cannot be approximated; next-fit for bin packing, set covering).

Other possible topics (to be selected at the discretion of the instructor)**:**
- Non-bipartite matching (Tutte's and Lovász' matrix theorems, Edmonds' blossom algorithm, Edmonds-Gallai structure theorem, Chinese postman problems, planar max-cut and Christofides' TSP approximation algorithm).
- Integer programming (modeling, branch and bound, polyhedral approaches: cutting planes, valid inequalities)
- Scheduling problems (polynomial-time algorithms for one-machine scheduling: WSPT for $1 / r = 0 / \sum w_j c_j$, EDD for $1/ d / T_{max}$, LP-based 2-approximation algorithm for makespan minimization on unrelated parallel machines).
- Dynamic programming algorithms for hard problems (knapsack, traveling salesman problem)
- MAX-SAT (Johnson's approximation algorithm, proof via pseudo-Boolean optimization)
- Local search metaheuristics (main concepts: neighborhood structures, steepest descent – hill climbing, simulated annealing).
- Nonlinear programming (extremum of a function from R^n to R, gradient and the Hesse matrix, unconstrained minimization, gradient method, constrained minimization, transformation to the unconstrained problem, Karush-Kuhn-Tucker conditions).

2.13 Course name: OPTIMIZATION METHOD IN LOGISTICS

Number of ECTS credits: **6**

Content:
The basic areas of logistics systems.
Theoretical characteristics of logistics and distribution supply chains.
- Material flow.
- Information flow.
- Cash flow.

Major decisions on supply chains.
- Location.
- Production.
- Inventories.
- Transportation.

Linear and nonlinear programming.
Discrete optimization.
Construction algorithms.
The use of heuristics and metaheuristics.
Specific examples of the tasks in logistics and distribution supply chains.
- Warehousing and storage planning.
- Compiling of - preparation of transport units.
- Transportation - Carp (road, rail, ship)

2.14 Course name: PERMUTATION GROUPS

Number of ECTS credits: **6**

Content:
- group action.
- orbits and stabilizers.
- extensions to multiply transitive groups.
- primitivity and imprimitivity.
- permutation groups and graphs.
- graph automorphisms, vertex-transitive and Cayley graphs.
- graphs with a chosen degree of symmetry.
- permutation groups and designs.

2.15 Course name: GALOIS THEORY

Number of ECTS credits: **6**

Content:
- Classical formulas
- Polynomials and field theory
- Fundamental theorem on symmetric polynomials and discriminants
- Roots of unity and cyclotomic polynomials
- Solvability by radicals
- Basic elements of Galois theory. Automorphisms. Galois extensions.
- Fundamental theorem of Galois theory.
- Classical theorems by Abel, Galois, Gauss, Kronecker, Lagrange, and Ruffini.
- Applications and examples.

2.16 Course name: SYMMETRIC-KEY CRIPTOGRAPHY

Number of ECTS credits: **6**

Content:
- hystory of the classical symmetric key encryption schemes
- fundamental concepts in the design of block and stream ciphers,
- modes of operation of symmetric key ciphers,
- cryptographic criteria for encryption schemes,
- security evaluation and generic attacks,
- basic building blocks of symmetric key encryption schemes,
- State-of-art ciphers and their security

2.17 Course name: GRAPH THEORY

Number of ECTS credits: **6**

Content:
- **Fundamental concepts.** What is a graph? Paths, cycles, and trails. Vertex degrees and counting. Directed graphs.
- **Trees and distances.** Basic properties. Spanning trees and enumeration. Trees and optimization.
- **Matchings and factors.** Matchings and covers. Algorithms and applications. Matchings in general graphs.
- **Connectivity and paths.** Cuts and connectivity. k-connected graphs. Flows in networks.
- **Coloring of graphs.** Vertex colorings and upper bounds. Chordal graphs and perfect graphs.
- **Edges and cycles.** Line graphs and edge colorings. Hamiltonian cycles.
- **Planar graphs.** Embeddings and Euler's formula. Characterizations of planar graphs.

2.18 Course name: CODING THEORY

Number of ECTS credits: **6**

Content:
- mathematical background (groups, rings, ideals, vector spaces, finite fields);
- basic concepts in coding theory;
- algebraic methods for the construction of error correcting codes;
- Hamming codes;
- Linear codes;
- Binary Golay codes;
- Cyclic codes;
- BCH codes;
- Reed-Solomon codes;
- bounds (Hamming, Singleton, Johnson's bound , ...)

2.19 Course name: MEASURE THEORY

Number of ECTS credits: **6**

Content:
- The concept of measureability. σ-algebra of measurable sets. Measurable functions. Borel sets and Borel measurable functions. Measureability of limit functions. Simple functions.
- Integral of nonnegative measurable functions and complex measurable functions. Fatou's lemma. Lebesgue's monotone convergence theorem and Lebesgue's dominated convergence theorem. Sets with measure zero and the concept of equality almost everywhere. L^p spaces.
- Positive Borel measures. Support of a function. Riesz's representation theorem for positive linear functional on algebra of continuous functions with compact support. Regularity of Borelovih measures. Lebesgu's measure.
- Approximation of a measurable function with continuous function. Lusin's theorem.
- Complex measures. Total variation. Absolute continuity. Lebesgue-Radon-Nikodym's theorem. L^p spaces as reflexive Banach spaces.
- Measure differentiability, symmetrical derivative of a measure. Absolute continuous functions and fundamental theorem of calculus. Theorem on new variables in integration.
- Product measure and Fubini's theorem. Completion of product Lebesgue measures.

2.20 Course name: NUMBER THEORY

Number of ECTS credits: **6**

Content:
- Divisibility of numbers. Greatedst common divisor. Least common multiple. Euclid's algorithm.
- Prime numbers. Writing numbers in other bases.
- Divisibility criterions. Congruences. Theorems of Fermat and Euler.
- Solving congruence equations. Quadratic reciprocity law.
- Linear and quadratic diophantine equations. Continued fractions. Arithmetical functions.
- Möbius inversion formula.

2.21 Course name: TOPOLOGY

Number of ECTS credits: **6**

Content:
- *Topological Spaces*. Topological Structure on a Set. Continuous Mappings. Bases and Subbases. Separation Aksioms.
- *Compactness*. Definition of a Compactness. Compact Metric Spaces. Compact Subspaces. Mappings of Compact Spaces. Locally Compact Spaces.
- *Connectedness*. An Ordinary Connectedness and Connectedness with Paths. Components. Local Connectedness.
- *Products*. Topological Product of Finitely Many Factors. Topological Properties of Finite Products. Topological Product of Infinitely Many Factors.
- *Real-Valued Continuous Functions*. Existence and Extension of Functions. Stone-Weierstrass Theorem.
- *Quotient Spaces.* Quotient Topology. Mappings of Quotient Spaces. Gluing. Projective Spaces.
- *Fundamental Theorems of Topology of Euclidean Spaces.* Brouwer Fixed-Point Theorem. Jordan Theorem. Invariant Open Sets. Schönflies Theorem.

2.22 Course name: HISTORY AND PHILOSOPHY OF MATHEMATICS

Number of ECTS credits: **6**

Content:
- The history of the concept of number. Main and ordinal numbers in different languages. History of writing numbers: hieroglyphically, alphabetically, transition to positional (Chinese), positional. Algorithms, calculators.
- Number Theory - primes, Euclidean algorithm, Diophantine equations. Fractions, rational numbers. Roots, algebraic equations. The symbolism of algebra - unknowns.
- Famous design tasks. Pythagorean Theorem and related content. The number π. Famous curves. Trigonometry. Deductive method in mathematics.
- Rhind and Moscow Papyrus. Babylonian cuneiform script following Neugebauer. Ten classics (Suang-Ching). Euclid’s Elements. Archimedes' collected works. Bhaskara: Lilavati. Almagest. Fibonacci: Liber Abaci.
- A historical overview of computer science (from calculators to computing machines, from calculations to programs, from data to information, between mathematics and engineering).
- The history of mathematics in Slovenia (textbooks, scientific papers, e.g., Vega)
- Historical development of mathematical and meta-mathematical terms.

2.23 Course name: SEMINAR – INTRODUCTION TO RESEARCH WORK

Number of ECTS credits: **6**

Content:
The course consists of the most important research topics from the field of mathematics.

2.24 Course name: MATHEMATICAL TOPICS IN ENGLISH II

Number of ECTS credits: **6**

Content:
The most current research topics in the field of mathematics are taught, which, among others, may include the following topics:
- history of the concept of number,
- number theory,
- algebra,
- analysis,
- linear programming,
- famous construction problems,
- overview of the history of computing,
- history of Mathematics in Slovenia,
- historical development of mathematical concepts.

As INTERNALLY ELECTIVE COURSES students can also choose the following courses from the undergraduate study programme **Mathematics in Economics and Finance** at UP FAMNIT: Financial Mathematics, Game Theory, Stochastic Processes I, Fundamentals of Insurance.

Contents of the above mentioned courses are available in the presentation of the study programme (Course structure).